Effect of ferromagnetic contacts on spin accumulation in an all-metallic lateral spin-valve system: Semiclassical spin drift-diffusion equations

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We study the effect of the ferromagnetic (FM) contacts on the spin accumulation in the lateral spin-valve system for the collinear magnetization configurations. When an additional FM electrode is introduced in the all-metallic lateral spin-valve system, we find that the transresistance can be fractionally suppressed or very weakly influenced depending on the position of the additional FM electrode, and relative magnitudes of contact resistance and the bulk resistance defined over the spin-diffusion length. Nonlocal spin signals such as nonlocal voltage drop and leakage spin currents are independent of the magnetization orientation of the additional FM electrode. Even when the additional contact is nonmagnetic, nonlocal spin signals can be changed by the spin current leaking into the nonmagnetic electrode.

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I. INTRODUCTION

Electrons are characterized by their quantized spin and charge. In conventional electronic devices, only the charge degree of freedom has been employed for the control of the electron transport. A new field of spintronics[1] was born from both experimental and theoretical efforts to harness the electron's spin degree of freedom in order to control the electric current in the devices. One of typical spintronic devices is the spin valve which is a hybrid structure of ferromagnetic (FM) metal/nonmagnetic (NM) material/FM metal. The current passing through the spin valve depends on the magnetization configuration of two FM metals. In the collinear case, usually more current flows through the spin valve in the parallel configuration than in the antiparallel configuration. The difference in resistance between the two is called magnetoresistance (MR). In the noncollinear case or when two magnetization orientations are neither parallel nor antiparallel, the spin-polarized current from one FM electrode exerts the spin torque[2–4] on the other FM electrode and induces the magnetization dynamics. Examples of a spin valve are giant magnetoresistance (GMR) devices,[5,6] magnetic tunnel junctions,[7] nanopillars,[8] etc.

In contrast to vertical spin valves, lateral spin valves are characterized by their multiterminal functionalities and so are more favorable for integration into semiconductor electronics. Due to increased spacing between terminals, efficient spin injection and detection have been an interesting issue. The spin injection and detection experiments in the two-terminal geometry are obscured by other effects such as anisotropic magnetoresistance, Hall effect, etc. This defect was overcome by adopting the nonlocal spin-valve geometry[9] similar to the schematic device structure in Fig. 1. The original spin-valve devices contain two FM electrodes (vs three FM electrodes in Fig. 1) contacting the nonmagnetic base electrode. In this lateral spin-valve system, the spin transport was clearly observed with Al wires[10] by spatially separating the spin current path from the charge current path and thereby removing other undesirable effects. The spin-polarized current flows from the left of N (base electrode) into F1. That is, spin-polarized electrons are injected from F1 into base electrode N and is drained to the left of N. Due to asymmetry of two spin states in FM, the number of injected spin-up and spin-down electrons is different. In addition to charge current in the left of N ($x<L_1$), diffusion of injected spins generates spin current flowing to left and right of N symmetrically. Pure spin current to the right of N was detected[9–11] with another FM electrode by measuring the spin-dependent nonlocal voltage drop. The nonlocal spin injection and detection technique was also used to observe[12,13] the (inverse) spin Hall effect in diffusive nonmagnetic metallic strips. In these experiments, the spatial separation of charge and spin currents as well as the efficient spin injection is essential to observing the charge Hall voltage induced by the spin current.

Recently experimental groups[14–17] studied the spin transport in the lateral spin valves with the three FM electrodes as shown in Fig. 1. F1 is the spin-injection electrode (F_{si}) as usual, while F2 and F3 are the nonlocal voltage probes located outside the charge current path. While the spin-polarized electrons are injected from F1 into N and are

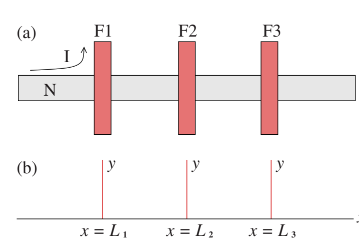

FIG. 1. (Color online) Schematic display of the lateral spin-valve system with three ferromagnetic electrodes. (a) Ferromagnetic electrodes are labeled as F1, F2, and F3 from left to right. The base electrode is denoted as N. Spins are injected from F1 to N by the spin-polarized current I flowing from the left end of N into F1. The electrode F1 is called the spin-injecting probe, while Fi with $i\neq 1$ will be called the nonlocal (voltage) probe in this paper. (b) The one-dimensional model geometry of the spin-valve system in (a).

drained into the left of N, the transresistance (TR) is measured between F1 and F2 or between F1 and F3. In the former case, F2 is the spin-detecting electrode F_{sd} and F3 is the additional electrode F_a. In the latter case, the roles of F2 and F3 are switched. The important issue is how sensitively the transresistance is affected by F_a.

One experimental group[14] found that F_a can drain the spin current and thereby significantly suppress the transresistance. They concluded that such additional FM electrode is relevant to the spin injection and accumulation in the multiterminal lateral spin valves. On the other hand, another group[16] found that the transresistance is weakly affected by F_a even when the contact resistance between F_a and N is Ohmic. Moreover the transresistance was observed[16] to be independent of the magnetization orientation of F_a (parallel or antiparallel to that of F_{si}). They concluded that such additional FM electrodes are irrelevant to the spin injection and accumulation in the multiterminal lateral spin valves. The existing experimental results seem to be contradictory to each other.

Motivated by this experimental situation, we study theoretically the spin transport in the lateral spin valve with three ferromagnetic electrodes as schematically shown in Fig. 1. We adopt the semiclassical spin drift-diffusion (SDD) equation[9,18] for the one-dimensional device structure and study the mutual effect of FM electrodes on their nonlocal spin signals such as nonlocal voltage drop and leakage spin current. We find that the efficiency of the spin current leakage into FM electrodes depends on the relative magnitude of junction resistance and the bulk resistance [defined over spin-diffusion length (SDL)] in FM and N electrodes. The voltage drop in F_{sd} is proportional to its leakage spin current with the proportionality constant given by the *effective* spin resistance which depends on the magnetization orientation of F_{sd}. Nonlocal spin signals are sensitive to the position of F_a relative to the positions of F_{si} and F_{sd}. When F_a is located in between F_{si} and F_{sd}, the transresistance can be either significantly or weakly affected by F_a depending on the relative magnitude of junction resistance and spin resistance. The effect of F_a is weak when F_a is located outside the region between F_{si} and F_{sd}. Even though the magnitude of the spin current and nonlocal spin signals may be modified by F_a, the flow direction of the spin current in the whole device is set by the magnetization orientation of F_{si} and so the nonlocal spin signals are independent of the magnetization orientation of F_a. This surprising result is already observed[16] in experiments and is the direct consequence of no charge current in F_a. Based on decoupling of charge and spin modes in the SDD equation and the Kirchhoff rules at the junction, we also show that the relationship between nonlocal spin signals and magnetization holds true even in realistic three-dimensional samples. These interesting properties, in fact, originate from zero charge current in nonlocal voltage electrodes. Irrelevance of magnetization orientation of additional FM electrode to nonlocal spin signals implies that even additional *nonmagnetic* electrode can modify nonlocal spin signals in the spin-detecting probe. Our theoretical study may be useful for clarifying the conflicting roles[14–17] of an additional FM electrode in the lateral spin-valve devices. In addition, our study is relevant to device applications because the multiterminal functionality is essential for device applications of lateral spin valves.

The rest of this paper is organized as follows. In Sec. II, the spin drift-diffusion equation is briefly introduced and the detailed algebras for the lateral spin valve with three FM electrodes are included. The results of our work for spin valves relevant to experiments are presented in Sec. III. In Sec. IV, our work is summarized and its relevance to experiments is discussed. Some algebraic details and interesting results are included in Appendixes A–C.

II. FORMALISM

From now on we are going to confine our discussion to the collinear magnetizations of three FM electrodes in spin valves and so we consider the spin-polarized transport in a steady state. The noncollinear magnetizations go beyond the scope of our paper since the current flow in the noncollinear magnetizations generates the spin transfer torque and induces the magnetization dynamics. In the collinear and diffusive transport, the spin drift-diffusion equations[9,18] have been very useful for understanding phenomenologically the spin-polarized transport in the spin-valve systems. Later the SDD equations were derived[19] from the semiclassical Boltzmann equation under the assumption that the SDL is larger than the mean-free path (MFP). Using the numerical solution of the spin-dependent Boltzmann equation, the validity of the SDD equations was further extended[20] to the case when the SDL is comparable to the MFP. The SDD equations have been widely used for analyzing the spin-injection experiments in various device geometries. The SDD formalism was also applied to the study of spin transfer torque[21–23] in the case of noncollinear magnetizations.

The SDD equations in the collinear magnetizations are written down for the spin-dependent electrochemical potential μ_α and electric current density $\mathbf{j}_\alpha$. Here $\alpha=\pm$ represents the spin-up (+) and spin-down (−) states. The presence of the spin-flip scattering in bulk mixes two spin states and the SDD equations can be written down in a matrix form,

$$\nabla^2\begin{pmatrix}\mu_+\\ \mu_-\end{pmatrix}=\begin{pmatrix}\dfrac{1}{D_+\tau_{+-}} & -\dfrac{1}{D_+\tau_{+-}}\\ -\dfrac{1}{D_-\tau_{-+}} & \dfrac{1}{D_-\tau_{-+}}\end{pmatrix}\begin{pmatrix}\mu_+\\ \mu_-\end{pmatrix}, \qquad (1)$$

$$\mathbf{j}_\alpha=\frac{\sigma_\alpha}{e}\nabla\mu_\alpha. \qquad (2)$$

Here D_α is the diffusion constant for spin direction $\alpha=\pm$ and τ_{+-} is the average spin-flip time for an electron from the spin direction + to −. σ_α is the conductivity for electrons with spin α and e is the absolute value of electron charge.

The matrix differential equation for the electrochemical potential can be solved[24] by analyzing the eigenvalues and eigenvectors of the matrix in the SDD equation. One eigenvalue is 0 and the corresponding eigenvector is $\binom{1}{1}$. The other eigenvalue defines the spin-diffusion length λ and is given by the expression

$$\frac{1}{\lambda^2} = \frac{1}{D_+\tau_{+-}} + \frac{1}{D_-\tau_{-+}}, \quad (3)$$

and its eigenvector is $\binom{\sigma_+^{-1}}{-\sigma_-^{-1}}$. To find this form of the eigenvector, the Einstein relation as well as the detailed balance relation is already invoked. Obviously the first eigenvector (charge mode) does not discriminate between two spin states, while the second one (spin mode) does.

In this section we analyze the spin-polarized transport in the spin-valve system based on the one-dimensional SDD equations. The device structure is displayed in Fig. 1(a), where the base electrode is contacted to three FM electrodes. Our primary goal is to understand the mutual influence of the ferromagnetic electrodes on the nonlocal spin signals such as the voltage drops and the leakage spin currents. FM leads are labeled as Fi with $i=1,2,3$ from left to right. The one-dimensional geometry, corresponding to the device structure, is displayed in Fig. 1(b), where the junctions between the base electrode and the FM leads are labeled as $x=L_i$ ($i=1,2,3$).

In experiments, the base electrode is nonmagnetic, but we are going to consider the case of magnetic base electrode with its nonzero bulk spin polarization (SP) β. Nonmagnetic case is recovered by a simple replacement $\beta=0$. The SP in each FM lead is denoted as β_i which is defined by the spin asymmetry in the spin-dependent conductivity $\sigma_{i\pm}$,

$$\beta_i = \frac{\sigma_{i+} - \sigma_{i-}}{\sigma_{i+} + \sigma_{i-}}. \quad (4)$$

With the total conductivity $\sigma_i=\sigma_{i+}+\sigma_{i-}$, the spin-up and spin-down conductivities can be written as

$$\sigma_{i\pm} = \frac{1}{2}(1 \pm \beta_i)\sigma_i. \quad (5)$$

For the base electrode, the spin polarization (β) in conductivity and the spin-dependent conductivities ($\sigma_\pm$) are defined in a similar manner.

When the spin-polarized electrons are injected from F1 into the base electrode N and are drained to left, the electrochemical potential in the FM leads can be written as

$$\frac{1}{e}\begin{pmatrix}\mu_{i+}\\ \mu_{i-}\end{pmatrix} = \left[\frac{I}{\sigma_1 A_1} y\,\delta_{i,1} - V_i\right]\begin{pmatrix}1\\1\end{pmatrix} - I_i\mathcal{R}_i e^{-y/\lambda_i}\begin{pmatrix}(1+\beta_i)^{-1}\\ -(1-\beta_i)^{-1}\end{pmatrix}. \quad (6)$$

Here $\delta_{i,1}$ is the Kronecker delta function. The spin-dependent current is determined by the equation

$$I_{i\alpha} = A_i \frac{\sigma_{i\alpha}}{e}\frac{d}{dy}\mu_{i\alpha}. \quad (7)$$

The charge current is given by the expression $I_{ci}=I_{i+}+I_{i-}=I\delta_{i,1}$ and flows only in F1 but not in Fi with $i\neq 1$. The ith FM lead is contacted to the base electrode at $x=L_i$. A_i, λ_i, σ_i, and β_i are the cross-sectional area, the spin-diffusion length, conductivity, and bulk spin polarization in conductivity of the ith FM lead, respectively. $\mathcal{R}_i$, defined by the relation

$$\mathcal{R}_i = \frac{\lambda_i}{\sigma_i A_i}, \quad (8)$$

is the resistance of the FM electrode over the spin-diffusion length. Due to an exponential decay of spin current, this definition of resistance makes sense physically when discussing the spin current. V_i is the voltage drop at each ferromagnetic electrode far away from the junction with the base electrode and is induced by the nonequilibrium spin injection and diffusion. Note that the common Fermi energy is dropped in writing the electrochemical potentials in this paper because the overall constant energy shift does not change the physics. The spin current in Fi $I_i^s=I_{i+}-I_{i-}$ is given by the expression

$$I_i^s = \beta_1 I\delta_{i,1} + I_i e^{-y/\lambda_i}. \quad (9)$$

The first term is the spin-polarized driving current, while the second comes from the spin accumulation and diffusion. Although no charge current flows in the region $x>L_1$, the spin current is induced in the base electrode due to the spin injection, accumulation, and diffusion. The spin current decays exponentially over the spin-diffusion length and in turn leaks into the other FM electrodes. I_i measures the magnitude of this leakage spin current at the interface between the base electrode and Fi. The leakage spin current also decays exponentially over the SDL in the FM electrodes. The set of six unknown parameters $\{V_i,I_i\}$ is to be determined by the Kirchhoff rules at the junctions. Note that V_i and I_i are null for $i\neq 1$ when the injected current is not spin polarized. V_1 can be nonzero for the tunneling barrier even when the injected current is not spin polarized. Hence we may call $\{V_i,I_i\}$ for $i\neq 1$ as the *nonlocal spin signals*.

In the common base electrode, we have the electrochemical potential for spin-up (+) and spin-down (−) electrons,

$$\frac{1}{e}\begin{pmatrix}\mu_+\\ \mu_-\end{pmatrix} = \frac{I}{\sigma A}(x-L_1)\,\theta(L_1-x)\begin{pmatrix}1\\1\end{pmatrix} - \sum_i J_i\mathcal{R}e^{-|x-L_i|/\lambda}\begin{pmatrix}(1+\beta)^{-1}\\ -(1-\beta)^{-1}\end{pmatrix}. \quad (10)$$

Here $\theta(x)$ is the step function and the spin-dependent current is computed from

$$I_\pm = \frac{A\sigma_\pm}{e}\frac{d\mu_\pm}{dx}. \quad (11)$$

A, λ, σ, and β are the cross-sectional area, the spin-diffusion length, conductivity, and bulk spin polarization in conductivity of the base electrode, respectively. $\mathcal{R}$, defined by the relation

$$\mathcal{R} = \frac{\lambda}{\sigma A}, \quad (12)$$

is the resistance over the spin-diffusion length in the base electrode. The charge current $I_c=I_++I_-=I\theta(L_1-x)$ flows only at the section $x<L_1$ and the spin current $I_s=I_+-I_-$ is given by the equation

$$I_N^s = \beta I \theta(L_1 - x) + \sum_i J_i \operatorname{sgn}(x - L_i) e^{-|x-L_i|/\lambda}. \quad (13)$$

The additional three unknown parameters $\{J_i\}$ are introduced for the base electrode. J_i represents the redistribution of spin current in the base electrode due to its leakage into the voltage probes (FM electrodes with $i=2,3$).

The electrochemical potentials are constructed such that the charge current is conserved at any junction in the device (charge continuity equation). No net charge current flows to the right of $x=L_1$ and the currents for spin-up and spin-down electrons are equal in their magnitude and opposite in their sign or flowing direction. This symmetry in spin current is strictly obeyed in the nonlocal spin-valve geometry even in the presence of the multiple ferromagnetic electrodes to the right of the spin-injecting FM electrode F1. Obviously the spin currents I_i and J_i are modified by the presence of other FM electrodes, which is our main research interest in this work.

There are nine unknown parameters $\{V_i, I_i, J_i\}$ ($i=1,2,3$), which should be determined by the boundary conditions or Kirchhoff rules at the junctions. As already mentioned above, the electrochemical potentials are constructed such that the charge current is conserved. In our work, the spin-flip scattering is neglected at the interface but is taken into account in bulks. In this case, the spin current is conserved at each junction and the following relations are obtained:

$$J_i = \frac{1}{2}(\beta - \beta_1) I \delta_{i,1} - \frac{1}{2} I_i. \quad (14)$$

We consider the case of dirty interface between the ferromagnetic electrodes and the base electrode. Due to a finite value of resistance at the interface, the electrochemical potential across the junction is not continuous and should be determined by Ohm's law,[19,24,25]

$$\frac{1}{e}\Delta\mu_{i\alpha} = I_{i\alpha}(y=0^+)\mathcal{R}_{ti\alpha}. \quad (15)$$

Here $\Delta\mu_{i\alpha}$ is the difference of the electrochemical potentials at $x=L_i$ between the base electrode and the Fi electrode. $\mathcal{R}_{ti\pm}$ is the spin-dependent junction resistance between the base electrode and Fi and is defined in terms of the spin polarization γ_i of junction resistance,

$$\mathcal{R}_{ti\pm} = \frac{2\mathcal{R}_{ti}}{1 \pm \gamma_i}. \quad (16)$$

$\mathcal{R}_{ti}$ is the total junction resistance or $\mathcal{R}_{ti}=\mathcal{R}_{ti+}\mathcal{R}_{ti-}/(\mathcal{R}_{ti+}+\mathcal{R}_{ti-})$. The clean or transparent contact can be recovered by a simple replacement $\mathcal{R}_{ti}=0$.

For the sign of β's (spin polarization), we are going to adopt the following convention. When the spin-up (spin-down) electrons belong to the majority (minority) channel at the Fermi level, the sign of β's is positive. On the other hand, the sign of β's is negative when the spin-up (spin-down) electrons belong to the minority (majority) channel. According to our convention, the sign of β's is reversed under the magnetization reversal. The same convention applies to the sign of γ's which are introduced to define the spin polarization in the resistance of the interface.

After some algebra as detailed in Appendix A, we find the expressions for I_i, J_i, and V_i, which contain all the information about the spin-polarized transport in the one-dimensional spin valve,

$$\frac{I_i}{I} = -(\beta_1 - \beta)\delta_{i,1} + G_{i1}[(\beta_1 - \beta)R_1 + (\gamma_1 - \beta)R_{t1}], \quad (17)$$

$$\frac{J_i}{I} = -\frac{1}{2}G_{i1}[(\beta_1 - \beta)R_1 + (\gamma_1 - \beta)R_{t1}], \quad (18)$$

$$\frac{V_i}{I} = -[(\beta - \beta_1)^2 R_1 + (\beta^2 - 2\beta\gamma_1 + 1)R_{t1}]\delta_{i,1} + [(\beta_i - \beta)R_i + (\gamma_i - \beta)R_{ti}]G_{i1}[(\beta_1 - \beta)R_1 + (\gamma_1 - \beta)R_{t1}]. \quad (19)$$

G_{ij} is the element of the matrix $\mathbf{G}$ defined in Eq. (A12) of Appendix A and has the dimension of conductance. The other set of material parameters, R_i and R_{ti}, is introduced in Appendix A and their definitions are repeated here for readers,

$$R_i \equiv \frac{\mathcal{R}_i}{1 - \beta_i^2}, \quad R \equiv \frac{\mathcal{R}}{1 - \beta^2}, \quad R_{ti} \equiv \frac{\mathcal{R}_{ti}}{1 - \gamma_i^2}. \quad (20)$$

These material parameters need our special attention. They have the dimension of resistance and deserve their own terminology. They are already called the *spin resistance* in the literature. First of all, the spin resistance is introduced to simplify the algebra as shown in Appendix A. As the above equations show, this spin resistance determines the nonlocal spin signals such as the voltage drops and the leakage spin currents in the voltage probes. More physical insights on the spin resistance are elaborated on in Appendix B.

Since we are interested in the nonlocal transport measurements, we focus on the leakage spin currents and voltage drops in the voltage probes (Fi with $i \neq 1$). The leakage spin current in the nonlocal voltage probes is given by the expression [Eq. (17)]

$$I_i = G_{i1}[(\beta_1 - \beta)R_1 + (\gamma_1 - \beta)R_{t1}]I. \quad (21)$$

This relation for the nonlocal spin current I_i suggests that the conductance matrix $\mathbf{G}$ contains all the information about the mutual effect of nonlocal voltage probes. Since the conductance G_{i1} does not depend on the magnetization configuration of the FM electrodes, the leaking spin current does not depend on the magnetization orientation of voltage probes but instead depends on the magnetization orientation of the spin-injecting FM electrode (F1) and the base electrode (if ferromagnetic, $\beta \neq 0$). This important observation can be understood as follows. The flow direction of spin current (the sign of I_i), in the base electrode as well as in the FM electrodes, is obviously determined by the magnetization configuration in the spin-injecting electrode. This means that the flow direction of spin current cannot be altered by the change in magnetic configurations in nonlocal voltage probes. This is due to the fact that the nonequilibrium spin current is generated by the spin-injecting electrode and not by nonlocal voltage probes. Another important observation is that the magnitude of spin current or I_i is not modified under the magnetization reversal of nonlocal voltage probes, which in fact derives from the symmetry in the SDD equations. This property of I_i derives from decoupling of spin and charge modes in SDD equations as well as the zero charge current in nonlocal voltage probes. Detailed analysis can be found in Appendix C. The relation $J_i=-I_i/2$ simply reflects the conservation of spin current at the interface between the voltage probe and the base electrode.

V_1^s defined below is ubiquitous in the expressions of I_i and V_i,

$$V_1^s=[(\beta_1-\beta)R_1+(\gamma_1-\beta)R_{t1}]I. \tag{22}$$

We may call V_1^s as the *spin potential* which is the source from the spin-injecting electrode and drives the spin current in the spin valve. The leakage spin current in nonlocal probes can be written as

$$I_i=G_{i1}V_1^s. \tag{23}$$

The spin current in the spin-valve device can be expressed in terms of the spin potential V_1^s and the conductance matrix $\mathbf{G}$ as

$$I_N^s=\beta I\theta(L_1-x)-\frac{V_1^s}{2}\sum_i G_{i1}\,\mathrm{sgn}(x-L_i)e^{-|x-L_i|/\lambda}, \tag{24}$$

$$I_1^s=\beta_1 I(1-e^{-y/\lambda_1})+[G_{11}V_1^s+\beta I]e^{-y/\lambda_1}, \tag{25}$$

$$I_i^s=G_{i1}V_1^s e^{-y/\lambda_i}, \quad i\neq 1. \tag{26}$$

The voltage drop in the nonlocal voltage probes is given by the expression [Eq. (19)]

$$V_i=[(\beta_i-\beta)R_i+(\gamma_i-\beta)R_{ti}]G_{i1}[(\beta_1-\beta)R_1+(\gamma_1-\beta)R_{t1}]I. \tag{27}$$

Note that the voltage drop can be written as the product of spin current and some sort of spin resistance as

$$V_i=[(\beta_i-\beta)R_i+(\gamma_i-\beta)R_{ti}]I_i. \tag{28}$$

V_i is the effective measure of weighted averaging of the spin-up and spin-down electrochemical potentials in Fi and so depends on the magnetization configuration of Fi. In addition, V_i can be understood as a shift in the Fermi level in order to satisfy the condition of zero charge current in the nonlocal voltage probes (see Appendix B for details). Although the leakage spin current I_i in Fi is independent of the magnetization orientations of all nonlocal voltage probes (parallel or antiparallel to that of spin-injecting probe), the voltage drop V_i depends on the magnetization orientation of Fi, the spin-injecting probe, and the base electrode but not on that of other voltage probes.

III. RESULTS

In this section our discussion is confined to the spin-valve system with three ferromagnetic electrodes and nonmagnetic ($\beta=0$) base electrode. The current I is injected from the left of nonmagnetic base electrode and is drained to F1 (see Fig. 1). Although the charge current is null to the right of the contact between F1 and the base electrode, the finite spin current is induced everywhere by the spin injection, accumulation, and diffusion. With two nonlocal FM electrodes (labeled as F2 and F3) contacted with the nonmagnetic base electrode to the right of F1, we want to study the mutual influence of two nonlocal FM electrodes on their voltage drops and leakage spin currents or nonlocal spin signals.

With three FM electrodes, the dimension of conductance matrix $\mathbf{G}$ is 3×3,

$$\mathbf{G}^{-1}=\begin{pmatrix} r_1 & \frac{1}{2}Rf_3 & \frac{1}{2}Rf_2 \\ \frac{1}{2}Rf_3 & r_2 & \frac{1}{2}Rf_1 \\ \frac{1}{2}Rf_2 & \frac{1}{2}Rf_1 & r_3 \end{pmatrix}. \tag{29}$$

Here $r_i=R_i+R_{ti}+\frac{1}{2}R$ for $i=1,2,3$ and $f_1=e^{-L_{23}/\lambda}$, $f_2=e^{-L_{13}/\lambda}$, and $f_3=e^{-L_{12}/\lambda}$, where $L_{ij}=|L_i-L_j|$ is the distance between the contacts of the ith and jth FM electrodes with the base electrode. After inserting the explicit expressions of G_{i1} into Eq. (21) with $\beta=0$ or

$$I_i=G_{i1}V_1^s, \quad V_1^s=(\beta_1R_1+\gamma_1R_{t1})I, \tag{30}$$

we find the leakage spin currents I_2 and I_3 in the nonlocal electrodes to be given by the expressions

$$I_2=-\frac{IR}{2D}e^{-L_{12}/\lambda}(\beta_1R_1+\gamma_1R_{t1})\left[R_3+R_{t3}+\frac{1}{2}(1-e^{-2L_{23}/\lambda})R\right], \tag{31}$$

$$I_3=-\frac{IR}{2D}e^{-L_{13}/\lambda}(\beta_1R_1+\gamma_1R_{t1})(R_2+R_{t2}). \tag{32}$$

Here D is the determinant of the matrix $\mathbf{G}^{-1}$ and is given by the expression

$$D=\prod_{i=1}^{3}\left(R_i+R_{ti}+\frac{1}{2}R\right)-\frac{1}{4}R^2\sum_{i=1}^{3}\left(R_i+R_{ti}+\frac{(-1)^{i+1}}{2}R\right)f_i^2. \tag{33}$$

Obviously the exponentially decaying factor can be extracted out as $G_{i1}=e^{-L_{1i}/\lambda}g_{i1}$ $(i=2,3)$, where g_{i1} is negative. As mentioned in Sec. II, the leaking spin current (I_2 and I_3) does not depend on the magnetization configuration of the nonlocal voltage probes (F2 and F3) but only on the magnetization configuration of the spin-injecting electrode (F1). This is clearly explained by the fact that the spin current is generated by the spin-injecting electrode.

The nonlocal voltage drops V_2 and V_3 are related to their leakage spin currents as

$$V_i=(\beta_iR_i+\gamma_iR_{ti})I_i, \quad i=2,3. \tag{34}$$

This relation suggests that the nonlocal voltage drop V_i is proportional to the corresponding nonlocal spin current I_i and the proportionality constant is the effective spin resistance which depends on the magnetization orientation of Fi. This resistance is the intrinsic material properties of the relevant FM electrode, so that the effect of the other FM electrode is completely embedded into the nonlocal spin current. This means that we can discuss the effect of multiple FM electrodes on transresistance in terms of the leakage spin current.

From Eqs. (31), (32), and (34), transresistances ($R_{s2}=V_2/I$ and $R_{s3}=V_3/I$) in F2 and F3 electrodes are reduced to the following forms:

$$R_{s2} = -\frac{R}{2D}e^{-L_{12}/\lambda}(\beta_1 R_1 + \gamma_1 R_{t1})(\beta_2 R_2 + \gamma_2 R_{t2}) \times \left[R_3 + R_{t3} + \frac{1}{2}(1 - e^{-2L_{23}/\lambda})R\right], \quad (35)$$

$$R_{s3} = -\frac{R}{2D}e^{-L_{13}/\lambda}(\beta_1 R_1 + \gamma_1 R_{t1})(R_2 + R_{t2})(\beta_3 R_3 + \gamma_3 R_{t3}). \quad (36)$$

We denote the transresistances in the absence of an additional FM electrode using the superscript as $R_{s2}^{(0)}$ and $R_{s3}^{(0)}$. The same notations with superscript will be used for nonlocal voltage drop and spin current. The effect of other nonlocal FM electrode on the transresistance can be quantified by computing the ratio $R_{si}/R_{si}^{(0)}$ with $i=2,3$. It follows from Eqs. (30) and (34) that

$$\frac{R_{si}}{R_{si}^{(0)}} = \frac{V_i}{V_i^{(0)}} = \frac{I_i}{I_i^{(0)}} = \frac{G_{i1}}{G_{i1}^{(0)}}. \quad (37)$$

The effect of an additional FM electrode on the transresistance can be measured by how much the nonlocal spin current is reduced or by the change in the conductance matrix **G** under the other FM electrode.

Two important facts can be read off from Eqs. (35) and (36). (i) The transresistance of one FM electrode (say F3) does not depend on magnetization orientation of the other electrode (say F2). That is, the transresistance R_{s3} in F3 does not depend on the spin polarization, β_2 and γ_2, of F2. In Ref. 16, the transresistance was observed to be independent of the magnetization orientation (parallel or antiparallel to F3) of the intervening FM electrode F2, which is supported by our theoretical results. However this fact does not necessarily mean[16] that the observed transresistance is not influenced by the additional FM electrode. The transresistance can be either significantly changed or weakly influenced by the presence of the additional intervening FM electrode, depending on sample and material parameters as we shall show below. (ii) The transresistance can be modified[15] even when the additional contacting electrode is nonmagnetic. The relative magnitude of the interface and bulk resistance (defined over the SDL) plays an important role in determining the transresistance. Irrespective of the magnetic or nonmagnetic nature of the intervening electrode, the transresistance will be influenced only by spin resistance and the interface quality.

According to Eq. (37), the effect of an additional FM electrode on the transresistance is equivalent to its effect on the nonlocal leakage spin current. After the spin current is injected from F1 electrode, it will flow into both directions in N and will leak into nonlocal probes. From this perspective we can expect that the effect will be much stronger when an additional FM electrode lies in between two (spin-injecting and spin-detecting) FM electrodes than when it lies outside two electrodes. In the former case, the nonlocal spin current in spin-detecting probe will be reduced proportionally by the amount of spin current drained into an intervening electrode. In the latter case, the injected spin current leaks into the spin-detecting probe first and then into an additional FM electrode, so that the effect will be weaker. Mathematically this difference between two cases comes from the asymmetry between Eqs. (31) and (32). Under the index exchange $2 \leftrightarrow 3$, I_2 and I_3 are inequivalent due to the additional term $(1-e^{-2L_{23}/\lambda})R/2$ in I_2.

To be more quantitative, let us consider I_2 when f_i^2's in Eq. (33) are all much less than unity. This is a good approximation in all-metallic lateral spin-valve systems since the spacing between the electrodes is comparable to the SDL which is on the order of few hundred nanometers. Under this approximation, we can readily show that

$$I_2 \simeq I_2^{(0)}. \quad (38)$$

Therefore, $R_{s2}=V_2/I$ is very weakly influenced by the FM electrode F3 and the transresistance is almost the same as that in the absence of the electrode F3. That is, the transresistance is not much changed by the additional electrode (F3) when it is contacted to the outside of F1 (spin current injected) and F2 (spin current detected). However, the effect of an additional FM electrode F3 cannot be neglected if F2 and F3 are closer to each other than the SDL. So much for this case.

We now focus on the case when an additional FM electrode lies in between the spin-injecting and spin-detecting electrodes. That is, we study the effect of F2 on the nonlocal spin signals for F3. In the absence of the intervening FM electrode F2, $R_{s3}^{(0)}$ is[25,26]

$$R_{s3}^{(0)} = -\frac{R}{2D_0}e^{-L_{13}/\lambda}(\beta_1 R_1 + \gamma_1 R_{t1})(\beta_3 R_3 + \gamma_3 R_{t3}), \quad (39)$$

$$D_0 = \left(R_1 + R_{t1} + \frac{1}{2}R\right)\left(R_3 + R_{t3} + \frac{1}{2}R\right) - \frac{1}{4}R^2 e^{-2L_{13}/\lambda}. \quad (40)$$

Note also that $R_{s3}^{(0)}$ can be obtained from R_{s3} by taking the limit $R_{t2} \to \infty$ or when the second intervening F2 is effectively decoupled from the nonmagnetic base electrode. The effect of the second intervening electrode F2 on the transresistance R_{s3} can be quantified by computing the ratio $R_{s3}/R_{s3}^{(0)}$,

$$\frac{R_{s3}}{R_{s3}^{(0)}} = \frac{D_0}{D}(R_2 + R_{t2}). \quad (41)$$

Below this general relation will be reduced to the simple forms case by case.

In order to provide some physical insights, let us consider the case when $f_i^2 \ll 1$. We find the simple form of $R_{s3}/R_{s3}^{(0)}$ as

$$\frac{R_{s3}}{R_{s3}^{(0)}} \simeq \frac{R_2 + R_{t2}}{R_2 + R_{t2} + \frac{1}{2}R}. \quad (42)$$

The reduction in R_{s3} stems from the leakage of spin current into the intervening electrode F2. The efficiency of spin leakage into F2 is quantified by the relative magnitude of the serial resistance R_2+R_{t2} in F2 and the resistance R in base electrode over the spin-diffusion length. We can understand qualitatively the results of Eq. (42) as follows. Spins are injected from F1 into the base electrode N and in turn diffuse into left and right of N. That is, the spin current flows in N and leaks into nonlocal probes. Just like charge transport, the spin current at the junction with F2 will continue to flow in N and also leak into F2. If the effective spin resistance R_2+R_{t2} of F2 is much larger than the spin resistance R of N, the leakage into F2 will be negligible and the spin current will mostly continue to flow in N. The leakage spin current I_2 is larger (smaller) if the effective spin resistance R_2+R_{t2} of F2 is smaller (larger) compared to the spin resistance R of N. Obviously the leakage into F2 reduces the spin current in N and in turn reduces the leakage spin current I_3. The larger (smaller) R_2+R_{t2} is, the larger (smaller) I_3 is.

The spin-diffusion length (SDL) is on the order of a few hundred nanometers in nonmagnetic metals and the SDL in FM metals is on the order of a few nanometers to a few tens of nanometers. The resistivity depends on the sample quality such as the impurities, defects, etc. Although SDL is 2 orders of magnitude different between FM and NM, the relative magnitude of resistance (R_F: ferromagnetic metal and R_N: nonmagnetic metal) defined over the SDL can be varied from device to device. Roughly $R_N \geq R_F$ in the spin-valve devices. Usually the interface between the FM electrodes and the nonmagnetic base electrode is Ohmic (R_t) but not in the tunneling regime. In real materials, we have the following order in resistance: $R_N \geq R_F > R_t$. For our theoretical study, we will consider both cases of Ohmic and tunneling interfaces, as well as other parameter regimes.

A. Clean F/N interface

We consider the clean interface between the base electrode and the FM electrodes, R_i, $R \gg R_{ti}$. To get the simple expression of R_{s3}, we take the limit $R_{ti}=0$. Suppose that the FM electrodes are the same material with roughly the same $R_i \simeq R_F$ for $i=1,2,3$. If the exponentially decaying factors (f_i^2) are negligible, we find the simple form of the transresistance ratio as

$$\frac{R_{s3}}{R_{s3}^{(0)}} \simeq \frac{2R_F}{2R_F + R}, \quad (43)$$

$$R_{s3}^{(0)} \approx -\beta_1\beta_3 \frac{2RR_F^2}{(2R_F+R)^2} e^{-L_{13}/\lambda}. \quad (44)$$

When $R_F \ll R$, the transresistance will be strongly suppressed by the additional intervening FM electrode. On the other hand, the transresistance will be fractionally reduced when R_F is comparable to R. In the other extreme case of $R_F \gg R$, the transresistance will not be affected by the intervening FM electrode.

As noted above, the transresistance can be affected by the nonmagnetic electrode, $\beta_2=0$ and $R_2=R_N$. Let us study this case in detail. In the clean limit of interface,

$$\frac{R_{s3}}{R_{s3}^{(0)}} \simeq \frac{2R_N}{2R_F + R}, \quad (45)$$

$$R_{s3}^{(0)} \approx -\beta_1\beta_3 \frac{2RR_F^2}{(2R_F+R)^2} e^{-L_{13}/\lambda}. \quad (46)$$

Since $R_N=\rho_2\lambda_2/A_2$ with $\beta_2=0$, we obtain the similar result as in the previous paragraph depending on the relative magnitude of R, R_N, and R_F. If the contacts between the base electrode and F1 and F3 are clean, but the contact with the intervening electrode F2 is in the tunneling regime, the effect of an additional electrode on the transresistance is negligible.

B. Tunneling F/N interface

When the junction resistance is dominant compared to the resistance over the spin-diffusion length in the FM lead and the base electrode or when $R_{ti} \gg R_j, R$, the expressions of the voltage drop [Eqs. (35) and (36)] are simplified as

$$\frac{V_2}{I} \approx -\frac{R}{2}\gamma_1\gamma_2 e^{-L_{12}/\lambda}, \quad (47)$$

$$\frac{V_3}{I} \approx -\frac{R}{2}\gamma_1\gamma_3 e^{-L_{13}/\lambda}. \quad (48)$$

The voltage drop at each junction is not influenced by the presence of the other FM leads when the junctions lie in the tunneling regime. In general, the expression of V_3 is not affected by the presence of the second FM lead (additional FM lead) as long as the contact is in the tunneling regime. When $R_{t2} \gg R_i$, $R_{s3}=R_{s3}^{(0)}$, so that the second FM lead is effectively disconnected from the base electrode.

When the accumulated spin is diffused efficiently into the second intervening FM lead, its effect may not be negligible. We still assume that the contacts with F1 and F3 lie in the tunneling regime. Let us see the extreme case of a transparent contact of F2 electrode to the base electrode. In this case, we may set $R_{t2}=0$ and the desired voltage drop is given by the expression

$$\frac{R_{s3}}{R_{s3}^{(0)}} = \frac{2R_2}{2R_2 + R}, \quad (49)$$

$$R_{s3}^{(0)} = -\frac{R}{2}\gamma_1\gamma_3 e^{-L_{13}/\lambda}. \quad (50)$$

That is, the transresistance can be changed by the second intervening electrode F2 if F2 is in clean contact with the base electrode or if spin leakage into F2 is efficient.

IV. DISCUSSION AND SUMMARY

Using the one-dimensional spin drift-diffusion equations, we studied theoretically the mutual effect of ferromagnetic electrodes on nonlocal spin signals (the leakage spin currents and the voltage drops) in the lateral spin valve with three ferromagnetic electrodes. We found the generic expression of the leakage spin current [Eq. (21)] and also a very simple relation [Eq. (28)] between the nonlocal voltage drop and the leakage spin current.

Equation (37) tells us that the effect of an additional electrode on the transresistance can be discussed in terms of the leakage spin current and in turn in terms of the conductance matrix. The measured nonlocal spin signals depend on the position of an additional FM electrode relative to the spin-injecting and spin-detecting electrodes. When the additional electrode lies outside the two FM electrodes, nonlocal spin signals are found to be weakly influenced due to the exponentially decaying spin coherence over the SDL. On the other hand, when it is located in between the two FM electrodes, the nonlocal spin signals can be strongly modified provided the junction resistance is lower than or comparable to the spin resistance defined over the spin-diffusion length in the FM electrodes and the nonmagnetic base electrode. If the junction resistance is high, the nonlocal spin signals are weakly modified even when the additional FM electrode is located in between the two FM electrodes The most general expression for the transresistance ratio is given by Eq. (41). In general, the nonlocal spin signal is not much modified when the additional electrode is in tunneling contact with the base electrode but is fractionally reduced when the contact is Ohmic. We also found that the nonlocal spin signals are independent of the magnetization orientation of the additional FM electrode, which agrees with the experimental observation.[16] This result suggests that even the intervening nonmagnetic electrode can change nonlocal spin signals, which was already observed[15] experimentally.

Since our theoretical study is based on the one-dimensional device structure, some care is needed when we try to apply our theoretical results to interpretation of experimental data. Strictly speaking, the experimental spin-valve structure is not one dimension in terms of the current distribution. Hamrle *et al.*[27] numerically showed that the nonlocal voltage drop depends strongly on the spatial distribution of the spin-polarized current. The one-dimensional approximation is valid when the current is uniformly distributed through the contact. When the contact is clean between the FM electrode and the base nonmagnetic electrode, the current flow may well not be uniform through the interface[16] and may be short circuited. In this case, the nonlocal spin signals may deviate from its theoretical estimate based on one-dimensional SDD equations. Keeping these restrictions in mind, let us apply our theoretical results to two experimental works.[14,16]

For numerical estimation ($R_s/R_{s0}=R_{s3}/R_{s3}^{(0)}$ in this section), we take examples of Co/Cu/Co and Py/Cu/Py lateral spin valves and use the following sample size and material parameters. The thickness and width of the nonmagnetic base electrode are taken as 80 and 300 nm, respectively. The width of all the ferromagnetic layers is assumed to be the same as 100 nm. The separation between nearest ferromagnetic layers is taken as 200 nm, which gives 300 nm of center-to-center distance. We use material parameters measured at low temperatures. The parameters for Cu are $1/\sigma_{Cu}=6\times10^{-9}$ Ω m (Ref. 28) and $\lambda_{Cu}=1$ μm.[10] For Co, we use $\beta_{Co}=0.46$,[28] $\gamma_{Co/Cu}=0.77$,[28] $1/\sigma_{Co}(1-\beta_{Co}^2)=7.5\times10^{-8}$ Ω m,[28] $\lambda_{Co}=59$ nm,[29] and $R_{Co/Cu}A/(1-\gamma_{Co/Cu})^2=0.52\times10^{-15}$ Ω m^2.[28] For Py, we take $\beta_{Py}=0.73$,[28] $\gamma_{Py/Cu}=0.70$,[28] $1/\sigma_{Py}(1-\beta_{Py}^2)=15.9\times10^{-8}$ Ω m,[28] $\lambda_{Py}=5.5$ nm,[28] and $R_{Py/Cu}A/(1-\gamma_{Py/Cu})^2=0.54\times10^{-15}$ Ω m^2.[28]

For the Co/Cu/Co spin valve, $R/2=125$ mΩ, $R_2=150$ mΩ, and $R_{t2}=17$ mΩ are obtained. The estimated spin signal is reduced to the value $R_s/R_{s0}=0.68$ by the intervening F2 electrode. V_2/I is also reduced by a factor of 0.87 due to F3 electrode. For the Py/Cu/Py spin valve, we have $R/2=125$ mΩ, $R_2=29$ mΩ, and $R_{t2}=18$ mΩ. The reduced spin signal V_3/I by the F2 electrode is $R_s/R_{s0}=0.46$. V_2/I is reduced by a factor of 0.87 due to F3 electrode.

Since the SDL of Co is rather long, R_2 is comparable to $R/2$ in the Co/Cu/Co case and R_s/R_{s0} is large. Since, in the Py/Cu, $R/2$ is larger than R_2 and R_{t2}, R_s/R_{s0} is small. The rather significant reduction in estimated V_2/I in both cases stems from our choice of the long SDL of Cu at low temperatures. The long SDL means that the chemical-potential splitting between opposite spin directions, although exponentially decaying, remains significant up to the position of the F3 electrode. The significant leakage of spin currents into F3 results in reduction in the spin signal. At room temperature, the SDL of the base electrode (nonmagnetic metal) is a few hundred nanometers such that the reduction in V_2/I by the F3 electrode is only a few percents. For the experimental conditions in Refs. 14 and 16, $R/2$ is comparable to R_2+R_{t2} and we can estimate theoretical value of R_s/R_{s0} [Eq. (42)]; $R_s/R_{s0}\simeq0.5$ although the observed R_s/R_{s0} is smaller for Ref. 14 and is close to unity for Ref. 16.

As pointed out in Ref. 16, the contact between the Permalloy electrode and the base Ag wire is very clean and the point injection and detection of current is suggested. In this case, the current distribution in the devices may well be nonuniform, so that our one-dimensional theory cannot be straightforwardly applied. We believe that the nonuniform current distribution is the main reason why some of our theoretical estimates are in poor agreement with the results of Ref. 16. We may discuss the relevance of the device dimensionality based on the effective spin resistance. According to our theoretical analysis, nonlocal spin signals are determined by the relative magnitude of junction resistance and spin resistance in FM and NM electrodes. This relevant resistance is defined under the assumption that the current distribution is uniform in the device. When the current distribution is not uniform as in real devices, we may still be able to define the spin resistance using the effective cross-sectional area which is smaller than the geometrical cross section of the sample. Nonuniform current distribution tends to increase junction resistance as well as spin resistance and will modify the magnitude of nonlocal spin signals. This may be one of the reasons for the discrepancy between two experimental results.[14,16]

Nonlocal spin signals (the leakage spin current and the voltage drop) in one nonlocal FM electrode are shown not to depend on the magnetization orientation (parallel or antiparallel) of the other nonlocal FM electrode. We believe that this symmetry of nonlocal spin signals is robust against the sample dimensionality, although their magnitude is sensitive to samples. The spin current is generated by the spin-injecting FM electrode and so its flow direction cannot be changed by the magnetization orientation of nonlocal FM electrodes. In addition, the magnitude of spin current does not depend on the magnetization orientation of nonlocal FM electrodes. This property derives from both decoupling of spin and charge modes in the SDD equations and zero charge current in nonlocal voltage probes. Hence our conclusion about the relationship between nonlocal spin signals and magnetization in nonlocal voltage probes will not depend on the sample dimensionality and qualities. This point is demonstrated more explicitly in Appendix C.

Finally, we would like to discuss the properties of transresistance (TR) and (longitudinal) magnetoresistance (MR) in spin valves under magnetization reversal. Obviously, both TR and MR are modified under magnetization reversal of two probing FM electrodes. Under magnetization reversal, TR changes its sign while MR changes its value. Note that MR, in general, consists of the two contributions: one part (background) remains the same but the other changes its sign under magnetization reversal. Let us consider the effect of an additional FM electrode (F_a) on TR and MR. For the vertical spin valves, it will not be easy to implement F_a. So we consider TR and MR in the lateral spin valves with F_a. Usually MR is obscured by other effects in the lateral spin valves as mentioned before. However, with increased SDL, MR was successfully measured[30,31] in the carbon nanotube and graphenes. To measure MR in the lateral spin valves of Fig. 1, F1 and F3 are both current and voltage probes. Based on the results of Appendix C, we can argue that MR should be independent of the magnetization orientation of F2 (Ref. 32) because there is no charge current in F2 (an additional electrode). Explicit calculation,[33] using the SDD equations, confirms this claim. That is, both TR and MR are independent of the magnetization orientation of F_a. On the other hand, TR and MR depend on the magnetization orientation of ferromagnetic electrodes through which the charge current flows.

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APPENDIX A: DETAILS IN ALGEBRA

In this appendix we show that the algebraic manipulation can be highly simplified by a proper definition of material parameters and by the vector and matrix notations. Kirchhoff rules lead to the constraints given by Eqs. (14) and (15). Equation (15) can be written down explicitly leading to the following six relations ($i=1,2,3$):

$$\pm\sum_j \frac{J_j\mathcal{R}}{1\pm\beta}e^{-|L_i-L_j|/\lambda}-\left(V_i\pm\frac{I_i\mathcal{R}_i}{1\pm\beta_i}\right)=[(1\pm\beta_1)I\delta_{i,1}\pm I_i]\times\frac{\mathcal{R}_{ti}}{1\pm\gamma_i}. \qquad (A1)$$

For algebraic convenience, we introduce material parameters

$$R_i=\frac{\mathcal{R}_i}{1-\beta_i^2},\quad R=\frac{\mathcal{R}}{1-\beta^2},\quad R_{ti}=\frac{\mathcal{R}_{ti}}{1-\gamma_i^2}, \qquad (A2)$$

and $A_{ij}=e^{-|L_i-L_j|/\lambda}$. These material parameters highly simplify the complicated algebra as well as determine the spin currents in the nonlocal voltage probes. Then the voltage drop V_i can be written as

$$V_i=\mp I_iR_i(1\mp\beta_i)\pm(1\mp\beta)R\sum_j A_{ij}J_j-(1\mp\gamma_i)R_{ti}[(1\pm\beta_1)I\delta_{i,1}\pm I_i]. \qquad (A3)$$

Addition and difference of two V_i's lead to

$$V_i=-(1-\gamma_1\beta_1)R_{t1}I\delta_{i,1}+(\beta_iR_i+\gamma_iR_{ti})I_i-\beta R\sum_j A_{ij}J_j, \qquad (A4)$$

and

$$(R_i+R_{ti})I_i-R\sum_j A_{ij}J_j=(\gamma_1-\beta_1)R_{t1}I\delta_{i,1}. \qquad (A5)$$

It is much more convenient to introduce the matrix notation for the algebraic manipulation. $|I\rangle=(I_1I_2I_3)^t$, $|J\rangle=(J_1J_2J_3)^t$, $|V\rangle=(V_1V_2V_3)^t$, and $|1\rangle=(100)^t$. Here the superscript t represents the transpose of row vectors, so that its effect is to change them into column vectors. With these notations, Kirchhoff rules can be written in compact forms as

$$|V\rangle=-(1-\beta_1\gamma_1)R_{t1}I|1\rangle+[\hat{\beta}\mathbf{R}+\hat{\gamma}\mathbf{R}_t]|I\rangle-\beta R\mathbf{A}|J\rangle, \qquad (A6)$$

$$|J\rangle=-\frac{1}{2}|I\rangle+\frac{1}{2}(\beta-\beta_1)I|1\rangle, \qquad (A7)$$

$$0=(\mathbf{R}+\mathbf{R}_t)|I\rangle-R\mathbf{A}|J\rangle+(\beta_1-\gamma_1)R_{t1}I|1\rangle. \qquad (A8)$$

Here $\mathbf{R}$ and $\mathbf{R}_t$ are diagonal matrices with diagonal elements R_i and R_{ti}, respectively. Similarly, $\hat{\beta}$ and $\hat{\gamma}$ are diagonal matrices with diagonal elements representing the spin polarization of each FM electrode and the junction resistance, respectively. $\mathbf{A}$ is the matrix with its elements given by A_{ij}. Formally, the unknown parameters can be written in a more compact matrix form as

$$|I\rangle=(\beta-\beta_1)I|1\rangle+[(\beta_1-\beta)R_1+(\gamma_1-\beta)R_{t1}]I\mathbf{G}|1\rangle, \qquad (A9)$$

$$|J\rangle=-\frac{1}{2}[(\beta_1-\beta)R_1+(\gamma_1-\beta)R_{t1}]I\mathbf{G}|1\rangle, \qquad (A10)$$

$$|V\rangle=-[(\beta-\beta_1)^2R_1+(\beta^2-2\beta\gamma_1+1)R_{t1}]I|1\rangle+[(\beta_1-\beta)R_1+(\gamma_1-\beta)R_{t1}]\times[(\hat{\beta}-\beta)\mathbf{R}+(\hat{\gamma}-\beta)\mathbf{R}_t]I\mathbf{G}|1\rangle. \qquad (A11)$$

Here the matrix $\mathbf{G}$ is defined by the expression

$$\mathbf{G} = \left[\mathbf{R} + \mathbf{R}_t + \frac{1}{2}R\mathbf{A}\right]^{-1}. \tag{A12}$$

The matrix $\mathbf{G}$, with the dimension of conductance, is independent of magnetization configurations (parallel or antiparallel to the spin-injecting electrode F1) of the FM electrodes. The set of parameters, I_i, J_i, and V_i, contains all the information about the spin-polarized transport in nonlocal spin valves. In components, we have the spin currents

$$\frac{I_i}{I} = (\beta - \beta_1)\delta_{i,1} + G_{i1}[(\beta_1 - \beta)R_1 + (\gamma_1 - \beta)R_{t1}], \tag{A13}$$

$$\frac{J_i}{I} = -\frac{1}{2}G_{i1}[(\beta_1 - \beta)R_1 + (\gamma_1 - \beta)R_{t1}], \tag{A14}$$

and the voltage drops in the FM electrodes

$$\begin{aligned}\frac{V_i}{I} = &-[(\beta - \beta_1)^2 R_1 + (\beta^2 - 2\beta\gamma_1 + 1)R_{t1}]\delta_{i,1} \\ &+ [(\beta_i - \beta)R_i + (\gamma_i - \beta)R_{ti}]G_{i1} \\ &\times[(\beta_1 - \beta)R_1 + (\gamma_1 - \beta)R_{t1}].\end{aligned} \tag{A15}$$

Note that the final results are written down in a very compact form, using material parameters as well as the conductance matrix.

APPENDIX B: PHYSICAL MEANING OF SPIN RESISTANCE

Spin resistance was defined in order to simplify the algebra. In this appendix we are going to infuse some physical meaning into spin resistance. Let us recast Eq. (A1) for $i \neq 1$ (nonlocal voltage probes) into a more illuminating form as

$$\pm\frac{U_i}{1 \pm \beta} - V_i = \pm\frac{I_i}{2}\left(\frac{2\mathcal{R}_{ti}}{1 \pm \gamma_i} + \frac{2\mathcal{R}_i}{1 \pm \beta_i}\right), \tag{B1}$$

where U_i acts as the effective electric potential of the base electrode at the junction with Fi and is defined by

$$U_i = \mathcal{R}\sum_j J_j e^{-|L_i - L_j|/\lambda}. \tag{B2}$$

For electrons with negative charge, V_i is the electric potential for both spin directions far into Fi and $\pm U_i/(1 \pm \beta)$ is the electric potential for spin-up and spin-down electrons, respectively, of the base electrode at the junction with Fi. Refer to Eqs. (6) and (10). We can deduce that the current $I_i/2$ at the interface flows into (out of) Fi for spin-up (spin-down) electrons.

The left-hand side (LHS) of Eq. (B1) represents the electric potential difference for both spin directions between the base electrode and Fi at the deep inside. The right-hand side (RHS) is the product of the current $I_i/2$ and the effective spin-dependent resistance. The sign in front represents correctly the flowing direction of the spin-up and spin-down current, respectively. The first term in the parenthesis is the spin-dependent tunnel resistance as defined in Eq. (16). The second term is none other than the spin resistance, which was introduced in the main text. With this spin resistance, Eq. (B1) is the effective Ohm's law for the leakage spin-up and spin-down currents.

When U_i is eliminated from Eq. (B1), the relation between the nonlocal voltage drop V_i and the leakage spin current I_i or Eq. (28) is obtained. From Eq. (B1), we can deduce the physical meaning of V_i. No charge current flows in the nonlocal FM electrodes. V_i represents the shift of the electrochemical potential in Fi to satisfy the constraint of no charge current flow. If we eliminate V_i from Eq. (B1), we find the following relation:

$$I_i\left(R_{ti} + R_i + \frac{R}{2}\right) = R\sum_{j\neq i} J_j e^{-|L_i - L_j|/\lambda}. \tag{B3}$$

The material parameters are defined in Appendix A. How do we interpret this relation? This relation can be considered as the Ohm's law for the leakage spin current. The LHS is the product of the spin current I_i and the effective resistance. From the standpoint of Fi, the spin current I_i flows from both sides of the base electrode (R) through the junction (R_{ti}) and into Fi (R_i). Hence the effective resistance is $R/2 + R_{ti} + R_i$ as in the above equation. The RHS is the effective spin potential which combines the source term from F1 and the sink terms from other nonlocal FM electrodes.

APPENDIX C: DEPENDENCE ON MAGNETIZATION DIRECTIONS: THREE-DIMENSIONAL CASE

One of the key results of our paper is the independence of the spin accumulation, the spin current, and the nonlocal voltage on the magnetization directions of electrodes. This appendix is aimed to provide an insight into the origin of this independence in three-dimensional situations. We again consider the geometry in Fig. 1. Similar notations will be used. The three-dimensional SDD equation is given by Eq. (1) and the associated charge and spin current densities $\mathbf{j}^c, \mathbf{j}^s$ are given by

$$\mathbf{j}^c = \frac{1}{e}\nabla(\sigma_+\mu_+ + \sigma_-\mu_-), \tag{C1}$$

$$\mathbf{j}^s = \frac{1}{e}\nabla(\sigma_+\mu_+ - \sigma_-\mu_-). \tag{C2}$$

Similar relations hold for $\mu_{i\alpha}$, $\mathbf{j}_i^c$, and $\mathbf{j}_i^s$ in the Fi electrode. The system is subjected to the following boundary conditions. From the condition of no leakage current to air or insulating substrate,

$$\hat{\mathbf{n}} \cdot \nabla\mu_\alpha = \hat{\mathbf{n}}_i \cdot \nabla\mu_{i\alpha} = 0 \tag{C3}$$

should hold at the sample boundaries facing air or insulating substrate. Here $\hat{\mathbf{n}}$ and $\hat{\mathbf{n}}_i$ denote normal vectors perpendicular to the boundaries. From the constraint of the current continuity applied to the interface between the base electrode and the electrode Fi, one finds that the following relation should hold at the interface:

$$\hat{\mathbf{n}}\cdot\nabla\sigma_\alpha\mu_\alpha=\hat{\mathbf{n}}\cdot\nabla\sigma_{i\alpha}\mu_{i\alpha}. \tag{C4}$$

The Ohm's law provides another boundary condition for the interface,

$$\frac{1}{e}\Delta\mu_{i\alpha}=\mathcal{R}_{ti\alpha}\hat{\mathbf{n}}_i\cdot\nabla\frac{\sigma_{i\alpha}\mu_{i\alpha}}{e}. \tag{C5}$$

When combined with these boundary conditions, the SDD equation completely fixes the spin-dependent electrochemical potentials. Here we remark that the current density $\nabla(\sigma_{i\alpha}\mu_{i\alpha}/e)$, instead of the current $I_{i\alpha}$, appears in Eq. (C5) and thus $\mathcal{R}_{ti\alpha}$ in Eq. (C5) amounts to the spin-dependent junction resistance *per unit area* instead of the junction resistance. We also remark that the tunneling barrier at a junction may not be uniform in realistic experimental situations and such nonuniformity can be taken into account by simply regarding $\mathcal{R}_{ti\alpha}$ as a position-dependent quantity since Eq. (C5) remains still valid even for the nonuniform barrier as long as the tunneling current remains perpendicular to the interface.

In order to examine the dependence of the spin accumulation on the magnetization directions, we re-express the involved equations in terms of the spin accumulation $\mu_+-\mu_-$ and the charge potential $(\sigma_+\mu_++\sigma_-\mu_-)/\sigma$. The SDD equation [Eq. (1)] is again decomposed into the following two decoupled equations (spin mode and charge mode),[24]

$$\nabla^2(\mu_+-\mu_-)=\frac{1}{\lambda^2}(\mu_+-\mu_-), \tag{C6}$$

$$\nabla^2(\sigma_+\mu_++\sigma_-\mu_-)=0. \tag{C7}$$

The boundary conditions for the spin accumulation can be derived from Eq. (C3) and one obtains

$$\hat{\mathbf{n}}\cdot\nabla(\mu_+-\mu_-)=\hat{\mathbf{n}}_i\cdot\nabla(\mu_{i+}-\mu_{i-})=0. \tag{C8}$$

From Eq. (C4), one obtains

$$\hat{\mathbf{n}}\cdot\nabla(\mu_{i+}-\mu_{i-})=\frac{\sigma}{\sigma_i}\frac{1-\beta^2}{2}\left(\frac{1}{1+\beta_i}+\frac{1}{1-\beta_i}\right)\hat{\mathbf{n}}\cdot\nabla(\mu_+-\mu_-)+\frac{e}{\sigma_i}\left(\frac{1+\beta}{1+\beta_i}-\frac{1-\beta}{1-\beta_i}\right)\hat{\mathbf{n}}\cdot\mathbf{j}_i^c, \tag{C9}$$

and from Eq. (C5), one obtains

$$\frac{1}{e}\Delta(\mu_{i+}-\mu_{i-})=\frac{\sigma_i\mathcal{R}_{ti}(1-\beta_i^2)}{4e}\hat{\mathbf{n}}\cdot\nabla(\mu_{i+}-\mu_{i-})+\frac{\mathcal{R}_{ti}(\beta_i+\gamma_i)}{2}\hat{\mathbf{n}}\cdot\mathbf{j}_i^c. \tag{C10}$$

Now we are ready to discuss the magnetization direction dependence of the spin accumulation, which is completely fixed from its SDD equation [Eq. (C6)] and boundary conditions (C8)–(C10). Note that in these equations, all terms that depend on the magnetization directions are multiplied by the charge current density. Thus the spin accumulation should be independent of the magnetization directions of electrodes in which the charge current density vanishes.

Next we discuss the magnetization direction dependence of the spin current. The spin current can be obtained from the spin accumulation as follows:

$$\mathbf{j}^s=\frac{\sigma(1-\beta^2)}{2e}\nabla(\mu_+-\mu_-)+\beta\mathbf{j}^c, \tag{C11}$$

$$\mathbf{j}_i^s=\frac{\sigma_i(1-\beta_i^2)}{2e}\nabla(\mu_{i+}-\mu_{i-})+\beta\mathbf{j}_i^c. \tag{C12}$$

Then from the properties of the spin accumulation, it is evident that the spin current density should be independent of the magnetization directions of electrodes in which the charge current density vanishes.

Finally we discuss the charge potential, which is subjected to the SDD equation [Eq. (C7)]. The boundary conditions for the charge potential can be derived from Eq. (C3) and one obtains

$$\hat{\mathbf{n}}\cdot\nabla(\sigma_+\mu_++\sigma_-\mu_-)=\hat{\mathbf{n}}_i\cdot\nabla(\sigma_{i+}\mu_{i+}+\sigma_{i-}\mu_{i-})=0. \tag{C13}$$

From Eq. (C4), one obtains

$$\hat{\mathbf{n}}\cdot\nabla(\sigma_+\mu_++\sigma_-\mu_-)=\hat{\mathbf{n}}\cdot\nabla(\sigma_{i+}\mu_{i+}+\sigma_{i-}\mu_{i-}), \tag{C14}$$

and from Eq. (C5), one obtains

$$\frac{1}{e}\frac{\sigma_{i+}\mu_{i+}+\sigma_{i-}\mu_{i-}}{\sigma_i}-\frac{1}{e}\frac{\sigma_+\mu_++\sigma_-\mu_-}{\sigma}=\frac{\beta_i-\beta}{2e}(\mu_+-\mu_-)+\frac{\sigma_i\mathcal{R}_{ti}(1-\beta_i^2)}{16e}[(1+\beta_i)(1+\gamma_i)-(1-\beta_i)(1-\gamma_i)]\times\hat{\mathbf{n}}\cdot\nabla(\mu_{i+}-\mu_{i-})+\frac{\mathcal{R}_{ti}}{8}[(1+\beta_i)^2(1+\gamma_i)+(1-\beta_i)^2(1-\gamma_i)]\hat{\mathbf{n}}\cdot\mathbf{j}_i^c. \tag{C15}$$

Note that the SDD equation [Eq. (C7)] and the boundary conditions (C13) and (C14) are not dependent on the magnetization directions of any electrodes. Thus the magnetization direction dependence can arise only from the boundary condition (C15). From Eq. (C15) and from the properties of the spin accumulation, one then finds that the charge potential at the electrode Fi is independent of the magnetization direction of other non-current-carrying electrodes. This in turn implies that the nonlocal voltage measured between the electrode Fi and the base electrode ($x=+\infty$) should be independent of the magnetization directions of other non-current-carrying electrodes Fj ($j\neq i$).

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